A FRACTAL ORIENTED UPGRADING OF RELIABILITY AND SAFETY KNOWLEDGE OF REALISTIC CHEMICAL ENGINEERING PROBLEMS

Mirko DOHNAL

Department of Chemical Engineering, Technical University, 616 69 Brno

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Reliability and safety knowledge is very sparse. Its realistic and objective application is nearly impossible. Additional knowledge is needed to increase the reasoning power of reliability and safety knowledge bases. The conventional fractal analysis has been used for the study of chaos in physical systems. Its possible role in the evaluation of reliability/safety knowledge bases is studied in this paper. The only precondition for the application of fractal analysis is an ability to distinguish between specific and general knowledge items. This enables us to detect a level of inconsistency between mostly subjective additional knowledge items and the existing knowledge bases. The fractal analysis can characterise the knowledge base as one integrated complex. However, knowledge acquisition requires "local" evaluations as well. Therefore a discriminative analysis is used. A realistic man-machine dialogue (an evaluation of the mean time between failures of control valves) supported by the fractal and/or discriminative analysis is presented.

The RS (reliability and safety) knowledge acquisition is extremely time-consuming[1,2]. It is based on a heterogeneous knowledge structure[3]. Realistic RS decision making is an iterative process composed of many sub tasks. It is an ill-structured process and therefore its description is nearly always ad hoc[4].

In order to model complex RS systems effectivelly, all the available information must be used. Even very uncertain and subjective knowledge is valuable. It is the effectiveness with which uncertain/subjective knowledge is used which is very often the main distinction between good and bad models of the same system[5,6].

Primary information (knowledge) is such information which is available at the very beginning of its analysis. Re-used information (e.g. literature) supplies primary information provided that original observations are not accessible directly[7]. Re-used information is the most important and frequent type of industrial information item. Provided this primary knowledge (represented e.g. by tables of numerical values) is applied directly no expert system and its relatively powerful reasoning mechanism can be used[8,9].

A limited amount of a primary information set and information loss during the formal treatment can be significantly influenced by a suitable choice of pre-processing mode[10]. A pre-processing is the formal treatment of primary data in order to increase data addressability and instant applicability.

Addressability is a property of information item to be stored within a given information structure (e.g. data base, knowledge base). An instant applicability means a possibility to use the results of preprocessing for desired purposes without any further treatment.

KNOWLEDGE UPGRADING

The conventional RS records are the most important industrial RS knowledge[4]. They represent an artificially "mummified" skeleton of knowledge structure because of bad knowledge acquisition methods used to develop them[11].

The RS records (usually graphs, tables and simple shallow equations) are not suitable for an efficient uncertainty reasoning. An upgrading of these valuable records is a retrospective application of knowledge engineering algorithms to insert additional information (e.g. expert guess) into them to increase their reasoning power[12].

A skeleton of knowledge, when observed by an expert, always invokes a certain inspiration. This inspiration can represent a substantial increase in e.g. discriminative power of a future knowledge base. Therefore it is very important.

Complex RS knowledge is vague, sparse and inconsistent. Any real life situation will be described by more than one category of knowldge. Combining different types of knowledge requires that more detailed (accurate) knowledge be degraded to the level of the less accurate knowledge.

This is particularly severe when information is "shoehorned" into formalism chosen (e.g. statistical analysis) not for their good match to the structure of the knowledge but e.g. for computational efficiency and software availability. Any knowledge manipulation always involves a loss of information.

A general strategy for dealing with such knowledge is to modify it as little as possible thus reducing the information loss. This necessitates keeping primary knowledge intact as far as possible and making optimal use of all available formal tools. Therefore the broadest possible spectrum of formal calculi must be used.

The following uncertain calculi are likely to be used in RS models: probability[13], fuzzy[14] and rough[15] sets, qualitative[16] and order of magnitude[17] reasoning.

Industrial experience has proved that fuzzy models are easily understood, no special mathematical or formal knowledge is needed to take part in realistic upgrading activities and fuzzy models can be quickly modified. For details see e.g. refs[18,19].

INTEGRATED FRACTAL AND DISCRIMINATIVE ANALYSIS

There has for some time been a feeling that chaos, which is frequently observed in various physical and chemical phenomena, cannot be suitably described by traditional uncertainty calculi[20]. This was the basic motivation for the development of novel mathematical tools for dealing with chaotic phenomena[20,21].

Fractal analysis has already been used for the solution of some realistic problems which have a very close relation with reliability studies. An example is metal rupture[20,22].

There are many reasons why fractal analysis seems to be useful for the study of realistic physical phenomena[20 - 22]. However from the point of view of the application of fractal analysis to reliability knowledge bases, the most interesting is the possibility of using fractal analysis to characterise the chaos level in a knowledge base. Such an analysis may even take account of the mutual interrelationships between information items i.e. structure[23]. By "chaos" we mean, broadly, a lack of consistency, or coherence, in the knowledge base.

What is presented in Appendix 2 is a tailored view of multidimensional fractal analysis. It is not completely rigorous and consistent with the main body of "theoretical fractal analysis". However, it is computationally tractable, and allows an implementation to be developed suitable for handling multidimensional tasks that can be run on PC-compatible computers.

The basic idea of the new algorithm can be described by analogy to the gradual disappearance of detail in a picture when the picture is gradually moved away from an observer. The advantage of this new "fractal philosophy" is that it matches the requirements that were specified for the chaos analysis of realistic knowledge bases.

Very broadly, one can say that historically soft sciences (e.g. psychology), tended to be language oriented. Their methodology was largely phenomenological. Initially the soft sciences took over the formal tools which were traditionally used in the natural sciences and engineering[24,25].

However, the classical formal tools (e.g. sets of differential equations) developed for the natural sciences were not always applicable to the soft sciences. The main reason was the extreme vagueness and frequent inconsistencies in primary knowledge.

As a result, several new formal tools for dealing with uncertainties have been developed inside the soft sciences which embody many interesting ideas. It may not necessarily be obvious how to do so, but it is certainly worth considering how some of these ideas may be transferred into the natural sciences.

Naturally psychological and cognitive studies tend to be centred on a human or other animal subject. The transfer of techniques from the soft sciences into different scientific (engineering) branches will in general require the specification of some sort of engineering equivalent of the subject of a psychological experiment.

A severe problem which must be solved in all RS studies is an evaluation of knowledge base "reliability". Indirectly, it can be quantified by its "discriminative power". A simple cognitive problem will be used in order to demonstrate an approach to evaluation of the discriminative power using methods from soft sciences. A relatively simple cognitive problem is chosen; the mechanism of question answering. For details see Appendix 1.

An elementary knowledge of fuzzy set theory is needed. A sufficient engineering interpretation of this theory is given e.g. in refs[26,27].

A knowledge base K is represented by a set of conditional statements.

$$\begin{aligned}&\text{if } A_{1,1} \text{ and } \ldots A_{1,n} \text{ then } B_1 \text{ or}\\ &\text{if } A_{2,1} \text{ and } \ldots A_{2,n} \text{ then } B_2 \text{ or}\\ &\ldots\\ &\text{if } A_{m,1} \text{ and } \ldots A_{m,n} \text{ then } B_m\end{aligned} \qquad (1)$$

Fuzzy sets (see Fig. 1)

$$A_{i,j}, B_i \quad i = 1, 2, \ldots m, \quad j = 1, 2, \ldots n \qquad (2)$$

are one dimensional fuzzy sets.

As a part of knowledge acquisition activities a series of gradually modified knowledge bases is developed

$$K_1, K_2, \ldots K_g . \qquad (3)$$

Knowledge base K_V is a modification of knowledge base K_W. It means that additional knowledge A_K must be available for such modifications

$$\text{“}K_W + A_K \quad \rightarrow \quad K_V\text{”}, \quad V > W \quad \text{(see (3))} . \qquad (4)$$

Meta Heuristics

A reader who has no intention of going into detail can understand what follows if he intuitively accepts the explanations given in this chapter. These explanations are given as a set of meta heuristics[28]. For details see Appendix 1 and Appendix 2.

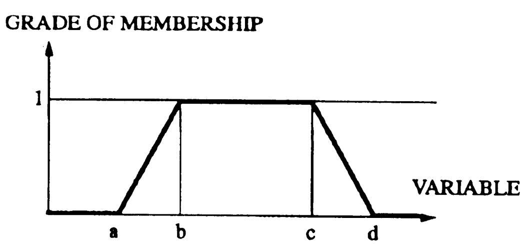

FIG. 1
Piecewise linear grade of membership

Fractal Analysis

– the level of chaos of the knowledge base K (*1*) can be quantified by fractional dimension D_K (see (*A 2.7*))

– if

$$D_V > D_W \quad \text{(see (}\mathit{4}\text{))} \tag{5}$$

then the additional knowledge A_K increased the level of chaos.

Discriminative Power

– the discriminative power of knowledge base K (1) is always related to a discriminative fuzzy set F (see Φ (*A 1.44*))

– the discriminative power P_F (K) of the fuzzy set F in connection with the knowledge base K can be characterized by point $P_{F,K}$ in the ternary diagram (see Fig. 2). The center of this triangle is I.

– if

$$IP_{F,V} > IP_{F,W} \tag{6}$$

then the discriminative power in relation to the fuzzy set F is decreased by the additional knowledge A_K (see (*4*)).

The fractal analysis (see Appendix 2)) is general in this sense, that it can characterize the knowledge base as one integrated system. The analysis of discriminative power is much more specific. Its specificity can be modified by a choice of discriminative sets (see Appendix 1).

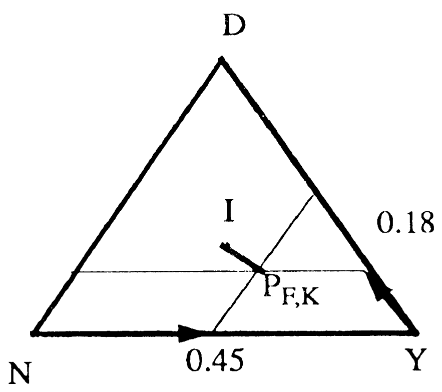

FIG. 2

Ternary diagram of discriminative power analysis, an example

KNOWLEDGE BASE UPGRADING

Any upgrading activity is ad hoc oriented. It is therefore not possible to present a generic upgrading algorithm. The below given heuristics have been tested by solving realistic problems. However they are, by no means, the best possible general variants.

– specify several sets of fuzzy sets A, B (see (2)) which are considered as good "dictionaries"

– evaluate fractional dimensions of all variants

– test the least chaotic variant using several meaningful discriminative sets

– if the discriminative powers are rather small then change the dictionaries (7)

– choose meaningful subsets of variables.

The very nature of the upgrading process eliminates any possibility of a formal description. Therefore a detailed description of the case study is presented.

CASE STUDY

The case study is based on an extensive analysis of control valves in chemical industry. The dependent variable (see B (2)) is the mean time between failures MTBF. The independent variables A (2) are

A_1 price, A_2 temperature, A_3 aggresivity, A_4 maintenance quality, A_5 type of motor, A_6 pressure drop, A_7 plant type.

The most important step towards good fuzzy knowledge base is the choice of all fuzzy sets A and B. This step is very subjective. Therefore the integrated fractal and discriminative analysis is used to minimize the level of subjectivity as much as possible.

Any upgrading process must start with the first knowledge base K_1 (3). The specifications of some one dimensional fuzzy sets A_{ij} (see (2)) which are used in K_1 are summarized in Table I.

The first 40 statements (m = 625, see (1)) are given in the following matrix (see Table II), where the first row of the matrix represents the following fuzzy simple conditional statement

$$\text{if } A_1 \text{ is CHE and } A_2 \text{ is COL and } A_3 \text{ is AGG and} \dots \text{then } B \text{ is } 0.2. \quad (8)$$

The knowledge base (Table II) is a concatination of two knowledge sub-bases

– set of general heuristics

– set of relatively accurate observations. (9)

The upgrading procedure starts with fractal analysis of both sub-bases. The following dictionaries give less chaotic knowledge base K_2 (see Table III).

The dictionaries for MTBF are not given. Any fuzzy specification of numerical value X given in Table II can be reconstructed:

$$a = 0.5\ X,\ b = c = X,\ d = 1.5\ X. \tag{10}$$

Much more transparent are two or three dimensional tasks. Therefore only a three dimensional tasks (see (7)) of the complete model (Table II) is studied:

$$\text{MTBF} = f(\text{price, temperature}).$$

Using standard statistical procedures the following estimation of the fractal dimension D is obtained from equation (*A 2.11*):

$$D = 2.04 \pm 0.106. \tag{11}$$

For details see refs[11,28].

TABLE I
Specifications of dimensional fuzzy sets A_{ij} (see Fig. 1)

Variable	Value	Abbreviation	*a*	*b*	*c*	*d*
A_1	cheap	CHE	0.1	0.3	0.5	0.8
	medium	MED	0.6	1.0	1.0	1.5
	expensive	EXP	1.0	3.0	3.0	6.0
A_2	cold	COL	−50	0	0	80
	warm	WAR	0	100	100	150
	hot	HOT	120	200	200	600
A_3	aggressive	AGG	1	1	1	1
	neutral	NEU	2	2	2	2
	medium	MED	0	1.5	1.5	3
A_4	no	NO	0	1	1	2
	standard	NOR	2	3	3	4
	excellent	EXC	3	4	4	5
A_5	hydraulic	HYD	1	1	1	1
	pneumatic	PNE	2	2	2	2
	electric	ELE	3	3	3	3
	positioner	POS	4	4	4	4
A_6	low	LOW	0.0	0.5	0.6	0.9
	high	HIG	0.8	1.0	1.0	1.3
	medium	MED	0.6	0.7	0.8	0.9
A_7	inorganic	ANO	1	1	1	1
	organic	ORG	2	2	2	2

TABLE II
Matrix of statements

Statement No.	Variable							
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	B
1	CHE	COL	AGG	NO	PNE	HIG	ANO	0.2
2	CHE	COL	AGG	NO	ELE	HIG	ORG	0.3
3	CHE	HOT	AGG	NO	ELE	HIG	ORG	0.3
4	CHE	HOT	AGG	NO	PNE	HIG	ANO	0.3
5	CHE	COL	AGG	NOR	ELE	HIG	ANO	0.3
6	CHE	COL	AGG	NO	ELE	HIG	ANO	0.3
7	CHE	HOT	AGG	NO	ELE	HIG	ANO	0.3
8	CHE	HOT	AGG	NO	HYD	HIG	ANO	0.3
9	CHE	COL	AGG	NO	PNE	HIG	ORG	0.4
10	CHE	COL	AGG	NO	HYD	HIG	ORG	0.4
11	CHE	COL	AGG	NOR	PNE	HIG	ANO	0.4
12	CHE	WAR	AGG	NO	PNE	HIG	ANO	0.4
13	CHE	COL	NEU	NO	ELE	HIG	ANO	0.4
14	CHE	WAR	AGG	NO	ELE	HIG	ANO	0.4
15	CHE	COL	AGG	NO	POS	HIG	ANO	0.4
16	CHE	HOT	AGG	NO	POS	HIG	ANO	0.4
17	CHE	WAR	AGG	NO	ELE	HIG	ORG	0.5
18	CHE	COL	AGG	NO	POS	HIG	ORG	0.5
19	CHE	COL	AGG	NO	PNE	LOW	ANO	0.5
20	CHE	COL	AGG	NO	ELE	LOW	ANO	0.5
21	CHE	COL	AGG	NO	POS	LOW	ANO	0.5
22	CHE	COL	NEU	NO	PNE	HIG	ANO	0.5
23	CHE	HOT	AGG	NOR	ELE	HIG	ANO	0.5
24	CHE	HOT	NEU	NO	ELE	HIG	ANO	0.5
25	MED	COL	AGG	NO	ELE	HIG	ANO	0.5
26	CHE	HOT	AGG	NO	HYD	HIG	ANO	0.5
27	CHE	COL	AGG	NO	ELE	HIG	ORG	0.6
28	CHE	HOT	AGG	NO	PNE	HIG	ORG	0.6
29	CHE	COL	AGG	NOR	ELE	HIG	ORG	0.6
30	CHE	COL	NEU	NO	ELE	HIG	ORG	0.6
31	MED	COL	AGG	NO	ELE	HIG	ORG	0.6
32	CHE	HOT	AGG	NO	HYD	HIG	ORG	0.6
33	CHE	HOT	AGG	NO	PNE	LOW	ANO	0.6
34	CHE	WAR	AGG	NO	ELE	LOW	ANO	0.6
35	CHE	HOT	AGG	NO	ELE	LOW	ANO	0.6
36	CHE	COL	AGG	NO	HYD	LOW	ANO	0.6
37	CHE	HOT	AGG	NOR	PNE	HIG	ANO	0.6
38	MED	HOT	AGG	NO	ELE	HIG	ANO	0.6
39	CHE	COL	AGG	NOR	HYD	HIG	ANO	0.6
40	CHE	COL	NEU	NO	HYD	HIG	ANO	0.6
	....	....	....	....	....	....	....	...
625	....	....	....	....	....	....	....	...

A simple fractal analysis of the two sub-bases (*9*) gives the following results:

$$\begin{aligned} \text{set of observations} \quad D_0 &= 1.0363 \pm 0.0047 \\ \text{heuristics} \quad D_h &= 2.2843 \pm 0.3701. \end{aligned} \quad (12)$$

So the observations (*9*) are much less chaotic than the heuristics:

$$D_h > D_0 \,. \quad (13)$$

From the point of reliability engineering, the dependent variable is the mean time between failures MTBF. Therefore the upgraded dictionary (*10*) is tested using the following five discriminative MTBF (Table IV).

The results are given in a graphic form in Fig. 3. The results for 1, 2, 4 are acceptable. The reason is the meta heuristic (*6*). The Fig. 3 indicates that there is "something wrong" with those statements in Table II which have such MTBFs which belong to

TABLE III
Knowledge base K_2 (see Fig. 1)

Variable	Value	Abbreviation	*a*	*b*	*c*	*d*
A_1	cheap	CHE	0.0	0.3	0.3	0.6
	medium	MED	0.3	1.0	1.0	2.0
	expensive	EXP	1.0	3.0	4.0	6.0
A_2	cold	COL	–50	0	0	100
	warm	WAR	0	100	100	200
	hot	HOT	150	200	200	450
A_3	aggressive	AGG	1	1	1	1
	neutral	NEU	2	2	2	2
	medium	MED	0	1.5	1.5	3
A_4	no	NO	0	1	1	2
	standard	NOR	2	3	3	4
	excellent	EXC	3	4	4	5
A_5	hydraulic	HYD	1	1	1	1
	pneumatic	PNE	2	2	2	2
	electric	ELE	3	3	3	3
	positioner	POS	4	4	4	4
A_6	low	LOW	0.0	0.5	0.6	0.9
	high	HIG	0.7	1.0	1.0	1.3
	medium	MED	0.6	0.7	0.7	0.8
A_7	inorganic	ANO	1	1	1	1
	organic	ORG	2	2	2	2

"medium" or "very long" (see Table IV and Fig. 3). This information partially identifies what must be changed. The new dictionaries are in Table V.

A realistic upgrading study requires several hundreds modifications[11,28]. It is therefore not possible to present the whole man-computer dialogue in detail.

CONCLUSION

In practice a RS engineer makes semi optimal decisions using his past experience which is not formalized at all[29]. Usually no severe mistakes are made provided more or less routine problems are solved. However rather often non-traditional decisions must be made[30].

Many different engineering activities (accident analysis, loss prevention) depend heavily (if not exclusively) on old records. Therefore the upgrading is inevitable.

TABLE IV
Five discriminative MTBF (see Figs 1 and 3)

MTBF	*a*	*b*	*c*	*d*	Point
Very short	0.0	0.3	0.4	0.8	1
Short	0.4	0.8	1.0	2.0	2
Medium	1.0	2.0	3.5	6.0	3
Long	3.5	6.0	7.0	14.0	4
Very long	6.0	14.0	20.0	40.0	5

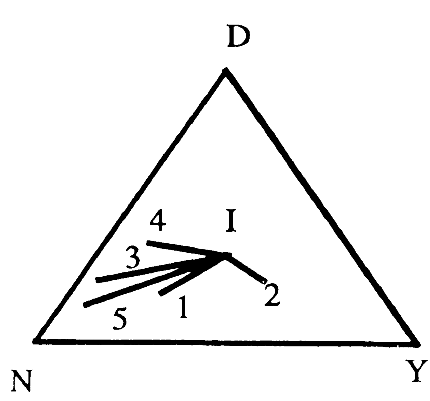

FIG. 3
Graphic representation of the discriminative power of five discriminative sets (*18*)

This work is at an early stage of development. It is rather novel approach to validate knowledge bases. However it has been used for several realistic cases. For example:

- high pressure vessel
  - crisp ruptures
- control valves
- life time of
  - ball bearings (vibrations)
  - rubber sealings
  - steam generators of nuclear power plants.

These realistic case studies demonstrate that new formal tools are needed to integrate objectively identifiable knowledge and subjective experience and analogy. It is not yet clear how to create the best network of calculi to upgrade knowledge. This structure is effected by the structure of available objective/subjective knowledge and by the expert

TABLE V
New dictionaries (see Fig. 1)

Variable	Value	Abbreviation	*a*	*b*	*c*	*d*
A_1	cheap	CHE	0.0	0.3	0.3	0.7
	medium	MED	0.3	1.0	1.2	2.0
	expensive	EXP	1.0	3.0	4.8	8.0
A_2	cold	COL	-50	0	0	100
	warm	WAR	0	100	100	200
	hot	HOT	150	200	200	450
A_3	aggressive	AGG	1	1	1	1
	neutral	NEU	2	2	2	2
	medium	MED	0	1	1.5	3
A_4	no	NO	0	1	1	2
	standard	NOR	2	3	3	4
	excellent	EXC	3	4	4	5
A_5	hydraulic	HYD	1	1	1	2
	pneumatic	PNE	1	2	2	2
	electric	ELE	3	3	3	3
	positioner	POS	4	4	4	4
A_6	low	LOW	0.0	0.3	0.5	0.9
	high	HIG	0.7	0.8	1.1	1.3
	medium	MED	0.5	0.7	0.6	0.8
A_7	inorganic	ANO	1	1	1	1
	organic	ORG	2	2	2	2

personality as well. This makes the development of upgrading theory much more complicated.

It would be highly desirable to formally merge the fractal analysis and the discrimination analysis. There is a relation between the fractal dimension D and the question stability.

It must be emphasized that the present level of knowledge does not allow elimination of human experts from any decision making. This decision making cannot be totally objective because of information shortage. Therefore, the only possibility is a symbiosis of human reasoning, and the high-speed data processing offered by computers.

APPENDIX 1

Discriminative Power

A simple example of a vague question is:

$$\text{IS THIS UNITE OPERATION RELIABLE?} \tag{A 1.1}$$

The answer that will be given to this question depends on the subjective understanding of what "all right" means. For simplicity, let us suppose that there are just two possible answers, namely YES and NO.

A human subject (expert or user) will have some knowledge concerning the evaluating of an attribute x of the system under study. The evaluation of an attribute is exclusively connected with the general understanding of the attribute x:

$$x > C_0 , \tag{A 1.2}$$

where C_0 is a cut off point.

A human expert will have a subjective perception of the cut off point C_0 of a question under study. Let the evaluation of the specific system attribute be S. Then the subject will answer YES if

$$S > C_0 \tag{A 1.3}$$

otherwise he/she will answer NO.

This simple example of a question answering mechanism is a simplification. One major limitation is that there is no mechanism for evaluating uncertainty in the answer. It would be much more realistic to allow for answers such as:

$$\text{VERY LIKELY YES .} \tag{A 1.4}$$

This requires some non traditional formalism to be developed to evaluate the uncertainty. One possible approach is to ask the same question twice to each of a number of experts[31]. The tested population (e.g. a set of experts) is, using the question under study, divided into three classes:

$$(\mathrm{YY}, \mathrm{NN}, \mathrm{C}), \qquad (A\ 1.5)$$

where YY is a set of those respondents who twice answered YES, NN twice NO, and C is a set of those that changed their answers.

The following concepts are used to characterize the question under study:

$$\begin{array}{l}\text{BALANCED–EXTREMAL}\\ \text{STABLE–VARIABLE}.\end{array} \qquad (A\ 1.6)$$

A question is stable if the cardinality of the set C (see (*A 1.5*)) is small in comparison with the cardinality of the sets YY and NN. A non-stable question is a variable question. A question is balanced if the cardinality of the sets YY and NN are approximately identical, otherwise it is extremal.

The cardinality of the set C

$$\text{car (C)} \equiv \text{a measure of question uncertainty} \qquad (A\ 1.7)$$

(see (*A 1.5*) may be used to qunatify the uncertainty in the question.

Let us suppose that the following set of fuzzy conditional statements

$$\begin{array}{l}\text{if } A_{1,1} \text{ and} \ldots \text{and } A_{1,n} \text{ then } B_1 \text{ or}\\ \text{if } A_{2,1} \text{ and} \ldots \text{and } A_{2,n} \text{ then } B_2 \text{ or}\\ \ldots\\ \text{if } A_{m,1} \text{ and} \ldots \text{and } A_{m,n} \text{ then } B_m\end{array} \qquad (A\ 1.8)$$

is a fuzzy model and its discriminative power is to be evaluated. Fuzzy sets (see Fig. 1)

$$A_{i,j}, B_i \quad i = 1, 2, \ldots m, \quad j = 1, 2, \ldots n \qquad (A\ 1.9)$$

are one dimensional (a piecewise linear grade of membership, see Fig. 1) and can be easily specified or/and modified using points *a*, *b*, *c*, *d*.

The Formal Analysis of an Answering Mechanism

Let us suppose that a question splits a set of s_n different objects into r different classes:

$$m_1, m_2, \ldots m_r$$

$$\mathrm{car}(m_1) + \ldots + \mathrm{car}(m_r) = s_n \,, \qquad (A\ 1.10)$$

where car(T) is the cardinality of set T.

A special case where $r = 3$ may be interpreted as a YES–NO–C question (see (*A 1.5*))

$$s_n = \mathrm{car(YY)} + \mathrm{car(NN)} + \mathrm{car(C)} \,. \qquad (A\ 1.11)$$

The number of objects is s_n. Therefore the total number of all possible pairs of objects is

$$s_n\,(s_n - 1)/2 \,. \qquad (A\ 1.12)$$

Two objects cannot be discriminated by the question if their answers belong to the same class m_i. The number of pairs of objects which belong to the i-th class is

$$m_i\,(m_i - 1)/2 \,. \qquad (A\ 1.13)$$

An index of internal discriminability I_i is (see ref.[32]):

$$I_i = 1 - m_i/s_n \,.$$

Let

$$I_{min} = \min\,(I_1, \ldots, I_r) \,.$$

The maximum value of index I_{min} is attained when

$$m_i/s_n = m_j/s_n \quad i, j = 1, 2, \ldots, r \,. \qquad (A\ 1.14)$$

Therefore the balanced question (see (*A 1.6*)) has the highest internal discriminability I_{min} because

$$\mathrm{car(YY)}/s_n = \mathrm{car(NN)}/s_n \,.$$

In what follows answers D

$$D = (\text{I DO NOT KNOW}) \qquad (A\ 1.15)$$

mean that the knowledge base (*A 1.8*) cannot answer a certain question. This type of answer substitutes type C if an expert system and rather than a human being answers a query.

Sets C (*A 1.5*) or D (*A 1.15*) are buffers between YES and NO answers. If this buffer is large then there is low risk that a YES answer is interpreted as a NO answer and vice versa. The cardinality of set D is used as a measure of uncertainty so then in same sense an adequate level of uncertainty is useful for discrimination (see (*A 1.7*)).

Consistency Test for a Fuzzy Knowledge Base

The following *n* knowledge bases are used for a consistency test:

$$KB_1, KB_2, \ldots, KB_m , \qquad (A\ 1.16)$$

where the *i*-th knowledge base KB_i is (compare with (*A 1.8*)):

$$\text{if } A_{1,1} \text{ and } \ldots \text{ and } A_{1,n} \text{ then } B_1 \text{ or}$$

$$\ldots$$

$$\text{if } A_{i-1,1} \text{ and } \ldots \text{ and } A_{i-1,n} \text{ then } B_{i-1} \text{ or}$$

$$\text{if } A_{i+1,1} \text{ and } \ldots \text{ and } A_{i+1,n} \text{ then } B_{i+1} \text{ or}$$

$$\text{if } A_{m,1} \text{ and } \ldots \text{ and } A_{m,n} \text{ then } B_m .$$

A fuzzy set

$$r_i = A_{i,1} \text{ and } \ldots \text{ and } A_{i,n} \qquad (A\ 1.17)$$

is submitted as a query. An answer of the knowledge base KB_i (*A 1.17*) is R_i

$$r_i \rightarrow KB_i\ (A\ 1.16) \rightarrow R_i . \qquad (A\ 1.18)$$

There are several algorithms for evaluating R_i (see e.g. refs[33,34]).

The complete consistency test gives a set of *n* fuzzy sets (answers):

$$R_1, R_2, \ldots, R_m . \qquad (A\ 1.19)$$

Fuzzification

Often, as a result of the sparseness of realistic knowledge bases in such fileds such as ecology, economics and engineering the following non fuzzy but absolutely vague answer is obtained:

$$(\mu_B(y) = 0 \quad y \in U) \equiv \text{I DO NOT KNOW (see } (A\ 1.15)) , \qquad (A\ 1.20)$$

where

$$\mu_B(y) \qquad (A\ 1.21)$$

is the grade of membership of y fuzzy set B and U is an universe of variable y.

In this case the fuzzification can be used. The fuzzification procedure increases the generality of the question r_i (see (*A 1.18*)); all its one-dimensional fuzzy sets $A_{i,j}$ are fuzzified as follows (see Fig. 1):

$$a - (b - a) f_i \quad \rightarrow \quad a$$

$$d + (d - c) f_i \quad \rightarrow \quad d , \qquad (A\ 1.22)$$

where f_i is the fuzzification coefficient of the i-th variable.

It is often the case that the first fuzzification is not sufficient. The I DO NOT KNOW answer (*A 1.20*) is obtained again. In this case another fuzzification is inevitable. However there must be an upper bound for a total number of fuzzifications. Since a question that is too fuzzy gives answers that are not relevant.

A very simple stop criterion is the total fuzzification coefficient F_T

$$F_T > u , \qquad (A\ 1.23)$$

where u is a current number of fuzifications.

A complementary stop criterion is the activation of a prescribed number of the conditional statements in knowledge the base (*A 1.8*).

The similarity of two m dimensional fuzzy sets V, W is

$$s(n, W, V) = s(n, V, W) = \min_{1 < j < n \quad X_j} (\max(\min(m_{Vj}(X_j), \mu_{Wj}(X_j)))) . \qquad (A\ 1.24)$$

The i-th statement is activated by the n-dimensional fuzzy query Q if the fuzzy set r_i, see (*A 1.18*), and Q are fuzzy similar i.e.

$$s(n, Q, r_i) > 0.$$

A set $w(Q)$ of those statements (*A 1.8*) which are activated by the query Q is:

$$w(Q) = \{i|\, s(n, r_i, Q) > t\}\,, \qquad (A\ 1.25)$$

where $t > 0$.

The ratio p indicates the level of knowledge base (*A 1.8*) activation:

$$p = \mathrm{car}(w(Q))/n\,. \qquad (A\ 1.26)$$

Provided the s-th statement is tested (as a part of the consistency test (*A 1.16*)) then

$$w(r_s) = \{i|\, s(n, r_s, r_i) > t, \quad i = 1, \ldots, m, s \neq i\} \quad F_T > uu_s \text{ (see } (A\ 1.23))$$

$$pp_s = \mathrm{car}(w(r_s))\,, \qquad (A\ 1.27)$$

where uu_s is the corresponding number of fuzzifications (see u in (*A 1.23*)).

There are three parameters which are chosen by the user, namely

$$p_t, t_u, F_T \quad \text{(see } (A\ 1.23)) \qquad (A\ 1.28)$$

p_t prescribed minimal value of fraction pp (see (*A 1.27*)) of activated statements

$$p_t < pp_s, \quad s = 1 \quad 1, \ldots, m\,,$$

t_u prescribed minimal value of the similarity, (see t in (*A 1.27*)) which is considered as significant.

Interpretation of Results

Answers R (see (*A 1.19*)) must be interpreted as elements of sets YY, NN and D. There are many possible algorithms, one of which is specified below.

The consistency test results R are used to evaluate the following similarities, see (*A 1.8*), (*A 1.19*):

$$s_i = s(1, R_i, B_i), \quad i = 1, 2, \ldots m\,. \qquad (A\ 1.29)$$

The fuzzy set R_i is interpreted (using s_i) as follows:

if

$$(p_t < pp_i \quad \text{for } t = t_u \quad \text{(see } (A\ 1.27, A\ 1.29))$$

and

$$F_T > uu_i \qquad \text{(see } (A\ 1.8, A\ 1.22, A\ 1.27)) \qquad (A\ 1.30)$$

then $R_i \in$ YY

$$\text{if } (p_t > pp_i \text{ or } F_T < uu_i) \quad (A\ 1.31)$$

then $R_i \in$ NN

$$\text{if (not (YY) and not (NN) then } R_i \in \text{D} . \quad (A\ 1.32)$$

Estimation of Knowledge Base Extremality and Stability

Normalized fractions *yy*, *nn* and *cc* are (see (*A 1.8*))

$$yy = \text{car(YY)}/m$$

$$nn = \text{car(NN)}/m$$

$$cc = \text{car(D)}/m \quad (A\ 1.33)$$

$$1 = yy + nn + cc . \quad (A\ 1.34)$$

A fuzzy interpretation of rules of thumbs requires a fuzzy specification of the following fuzzy relations:

LA larger; CL considerably larger; RE roughly equal. (*A 1.35*)

So then the following relation

$$(yy + nn)\ (\text{CL})\ cc$$

means that $yy + nn$ is considerably larger than *cc*.

A suitable definition of fuzzy set RE (see Fig. 4) is (notation is based on points *a*, *b*, *c* and *d* (see Fig. 1)):

$$d_{re} = -a_{re}$$

$$b_{re} = c_{re} = 0$$

$$d_{re} > 0 . \quad (A\ 1.36)$$

A definition of the fuzzy sets LA and CL (see Fig. 4) is

$$a_{la} = 0$$

$$b_{la} = c_{la} = d_{re}$$

$$d_{la} = b_{la} K$$

$$a_{cl} = b_{la}$$

$$b_{cl} = d_{la}$$

$$c_{cl} = d_{cl} = \infty$$

$$K > 1 . \qquad (A\ 1.37)$$

Only two constants are needed to specify the relations (*A 1.35*), namely K (see (*A 1.37*)) and point d_{re} (see (*A 1.37*), Figs 1, 4).

The following definitions are based on common sense. A stability coefficient is

$$S \equiv (yy + nn)\ (\mathrm{CL})\ (cc) \qquad (A\ 1.38)$$

which means that (see (*A 1.35*), Fig. 4)

$$S = \mu_{CL}\ (yy + nn - cc)\ . \qquad (A\ 1.39)$$

An extremality coefficient is

$$E = (yy\ (\mathrm{CL})\ nn) \text{ or } (nn\ (\mathrm{CL})\ yy)\ , \qquad (A\ 1.40)$$

where X or $Y = \max(X,Y)$.

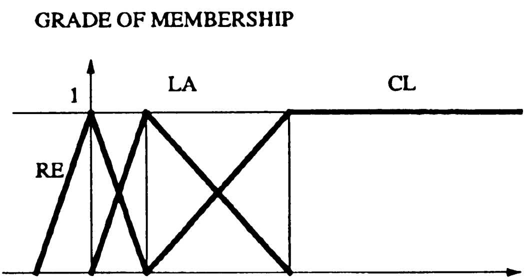

FIG. 4
Fuzzy specification of relations RE roughly equal, LA larger, CL considerably larger

A balanced coefficient B is

$$B = yy\,(\mathrm{RE})\,nn\,. \qquad (A\ 1.41)$$

A variable coefficient V is

$$V = (cc\,(\mathrm{LA})\,(yy + nn) \text{ or } (cc\,(\mathrm{CL})\,(yy + nn)\,. \qquad (A\ 1.42)$$

The fuzzy definitions of coefficients S, E, B and V must be indirectly based on the following common sense "feeling":

$$S = 1 - V$$

$$E = 1 - B\,. \qquad (A\ 1.43)$$

It indicates that e.g. an increase in stability is counterbalanced by a decrease in variability. This feeling is quantified by constants K, d_{re}, (see (*A 1.36*), (*A 1.37*)) and Eqs (*A 1.43*) are heuristics rather than equations.

Discriminative Sets

An engineer would be probably satisfied if he would know that a knowledge base is:

"good" (i.e. has a high discriminative power) for "low" MTBF
"not very good" for "medium" MTBF
"very unreliable" for "large" MTBF .

In this case three discriminative sets will be used, namely

"low MTBF", "medium MTBF" and "large MTBF" .

Therefore for engineering purposes an algorithm is needed which accepts not just answers YES, NO, D (do not know). This is why a fuzzy discriminative set (FCS) is introduced .

Every variable has its own set of fuzzy discriminative sets where $\Phi_{j,e}$ is e-th FCS for j-th variable (see (*A 1.8*)):

$$\Phi_{j,e} \quad j = 1, 2, \ldots, n; \quad e = 1, 2, \ldots z_j\,, \qquad (A\ 1.44)$$

where z_j is the number of FCSs of j-th variable.

The knowledge base (*A 1.8*) has $n + 1$ variables. The $(n + 1)$-th variable is a dependent variable. However any variable can be considered as a dependent variable. This comes from requirements of a realistic man-machine dialogue. Therefore a complex consistency test is represented by a matrix R (compare with (*A 1.19*)):

$$R_{i,j} \quad i = 1, \ldots, m; \quad j = 1, 2, \ldots, n + 1 \,. \qquad (A\ 1.45)$$

Let (see (*A 1.29*), (*A 1.44*), (*A 1.45*)):

$$v_{i,j,r} = s\,(1, R_{i,j}, \Phi_{j,r}) \;\; i = 1, \ldots, m; \;\; j = 1, \ldots, n + 1; \;\; r = 1, 2, \ldots, z_j \,, \qquad (A\ 1.46)$$

where i-th conditional statement, j-th variable as a dependent variable, r-th fuzzy cognitive set of the j-th variable.

Industrial applications very often require not only different dependent variables but different subsets of independent variables as well. A complete study of all possible subsets is prohibitively time-consuming. However any modest number of chosen subsets of independent variables can be analyzed by the same algorithm as the compete set of independent variables.

Cognitive Analysis Of Fuzzy Knowledge Base

Let us suppose that e.g. the FCS under study is

$$\text{FCS} = \text{medium MTBF} \qquad (A\ 1.47)$$

then if the corresponding element of the matrix R (*A 1.45*) is interpreted as NO then the complete interpretation actually means:

$$\text{NO medium MTBF} \,. \qquad (A\ 1.48)$$

Therefore cognitive YES, NO, D are not absolute but relative and they are always connected with a specific FCS.

The cognitive interpretation of $R_{i,j}$ in a connection with the specific FCS $\Phi_{j,e}$ (see (*A 1.46*)) is:

$$\text{if} \;\; (p_t < pp_i, F_T > uu_j, \;\text{and}\; v_{i,j,e} > t_u) \qquad (A\ 1.49)$$

then $yes_{i,j,e} = 1$ else $yes_{i,j,e} = 0$ (compare with (*A 1.30*))

$$\text{if} \;\; (p_t > pp_i \;\text{ for }\; t = 0 \;\text{ or }\; F_T < uu_i) \qquad (A\ 1.50)$$

then $no_{i,j,e} = 1$ else $no_{i,j,e} = 0$;

$$\text{if } yes_{i,j,e} = 0 \text{ and } no_{i,j,e} = 0 \text{ then } dd_{i,j,e} = 1 \text{ else } dd_{i,j,e} = 0 \,. \qquad (A\ 1.51)$$

The following simple formulae give cognitive discrimination coefficients $y_{j,r}$, $n_{j,r}$, $d_{j,r}$

$$y_{j,r} = (\sum_{i=1}^{m} \text{yes}\,(i,j,r))/m\,, \quad \text{see } (A\ 1.49) \qquad (A\ 1.52)$$

$$n_{j,r} = (\sum_{i=1}^{m} \text{no}\,(i,j,r))/m\,, \quad \text{see } (A\ 1.50) \qquad (A\ 1.53)$$

$$d_{j,r} = (\sum_{i=1}^{m} \text{no}\,(i,j,r))/m\,, \quad \text{see } (A\ 1.51)\,. \qquad (A\ 1.54)$$

Discriminative indexes indicate the discriminative power of all fuzzy cognitive sets of all variables.

THE INTERPRETATION OF DISCRIMINATIVE RESULTS

The knowledge base (*A 1.8*) can be stable (S), see (*A 1.38*), or variable (V), see (*A 1.42*). The second point of view is dichotomous categorization, namely balanced (B), see (*A 1.41*), or extremal (E), see (*A 1.40*). There are theoretically VB, VE, SB, SE variants, where e.g. VB means V and B.

VB YYYYYYYYYYYYYDDDDDDDDDDDDDDDNNNNNNNNNNN
SB YYYYYYYYYYYYYYYDDNNNNNNNNNNNNNNNNNNNNNN
SE YYDNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNN
SE YYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYYDNNNNN (*A 1.55*)

VE variant is practically impossible because many D answers together with many Y or N answers are mutually exclusive since in this case "many" is taken as relative to the total number of answers.

The Ternary Diagram and Cognitive Properties of Knowledge Basis

A ternary diagram is a common graphic representation of a chemical composition of a ternary mixture. Concentrations x, y, z of the mixture give:

$$x + y + z = 1\,. \qquad (A\ 1.56)$$

Eq. (*A 1.56*) represents a point in a ternary cognitive diagram *yy*, *nn*, *cc* (see (*A 1.52*) – (*A 1.54*)):

$$yy = y_{j,r}$$

$$nn = n_{j,r}$$

$$cc = d_{j,r} \,. \qquad (A\ 1.57)$$

Points N, Y, C in the ternary diagram are given as (see Fig. 5).

$$\begin{array}{lll} \text{N} & nn = 1 & cc = yy = 0 \\ \text{Y} & yy = 1 & cc = nn = 0 \\ \text{D} & cc = 1 & nn = yy = 0 \\ \text{I} & yy = nn = cc = 0.3333 & \end{array} \qquad (A\ 1.58)$$

Interval $D(T_1)$ (see Fig. 5) is a set of points
$yy = nn$.
Therefore interval $D(T_1)$ gives
B = 1 (see (*A 1.41*)).
The intervals
ND ($yy = 0$); DY ($nn = 0$)
give
B = 0.
Therefore E = 1 (see (*A 1.43*)).

The following matrix summarizes the numerical values of coefficients E, B, S, V (see Fig. 5).

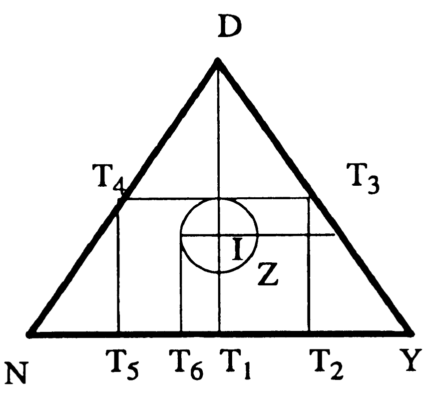

FIG. 5
Graphic description discriminative power of "good" knowledge bases

Coefficients	E	B	S	V	
N	1	0	1	0	
Point D	0	1	0	1	(*A 1.59*)
Y	1	0	1	0	
I	0.5	0.5	0.5	0.5	

Our industrial experience has proved that a set of constraints is needed to specify under which conditions certain theoretical recommendations are acceptable for industrial application[32,35].

E.g. point D is absolutely balanced (B = 1). The knowledge base (*A 1.8*) which by its properties corresponds to point D is absolutely balanced. The balanced knowledge base has the highest discrimination index (see (*A 1.14*)). However the fact that the knowledge base is balanced is not an advantage because its extremal variability (see (*A 1.59*)).

The balanced knowledge base is considered as well balanced if its ternary diagram point is in circle Z (see Fig. 5). Point I is the center of circle Z. Its radius is $(T_1)(T_6)$.

$$(T_1)(T_6) = 0.25$$

Any knowledge base that is not well balanced i. e. any point outside the circle Z is not very good for realistic discrimination.

Triangles $N(T_4)(T_5)$ and $Y(T_2)(T_3)$ (see Fig. 5) are sets of all points which are considered as not acceptably extremal from the point of view of discrimination. The rest of the triangle NDY is therefore practically stable. An example of a realistic result of the discriminative power is given in Fig. 4.

$$yy = 0.18$$

$$nn = 0.45$$

$$cc = 0.37 \qquad (A\ 1.60)$$

The absolute value of the interval $I - P_{F,K}$ (see Fig. 2)

$$\text{interval } IP_{F,K} \qquad (A\ 1.61)$$

represents the discriminative power of the set of conditional statements (*A 1.8*).

Optimization Algorithm

A general mathematical programming problem can be define as follows (see e.g. ref.[36]): minimize an objective function f:

$$\min f(X) \tag{A 1.62}$$

subjected to

$$H(X) = 0$$

$$G(X) \le 0 , \tag{A 1.63}$$

where

$$X = (X_1, X_2, \ldots, X_n) \tag{A 1.64}$$

is an n-dimensional vector. A solution is vector X_0:

$$f(X_0) = \min f(X)$$

$$H(X_0) = 0$$

$$G(X_0) \le 0 . \tag{A 1.65}$$

The problem of the best parameters evaluation (*A 1.28*) can be very easily transfer into an optimization problem by the following transformation

$$X_s \leftarrow p_t, t_u, F_T \tag{A 1.66}$$

and

$$f(X) \leftarrow IP_{F,K} . \tag{A 1.67}$$

APPENDIX 2

FRACTAL ANALYSIS

As a basic introduction to the ideas of fractal analysis, consider the following hypothetical study of the British coast length. Suppose that this coast was measured several times, each time using a different measuring unit from the sequence:

$$U_1, U_2, \ldots, U_v , \tag{A 2.1}$$

where

$$U_i < U_{i+1} \quad \text{for} \quad 1 \le i < v . \tag{A 2.2}$$

Let

$$L(U_s) \qquad (A\ 2.3)$$

be the total length of the sea coast if the s-th measuring unit U_s (see (*A 2.2*)) is used. A coastline has a characteristic roughness. Consequently as the scale of measuring unit decreases, so it will resolve more detail and hence measure a greater total length. In general, it can be demonstrated that for all i,

$$L(U_i) > L(U_{i+1})\ . \qquad (A\ 2.4)$$

A coast detail of characteristic dimension d, perhaps a circular stone for example, is certainly "detected" by the measuring unit U_s provided

$$U_s < d. \qquad (A\ 2.5)$$

In this case the stone contributes to the total coast length. Whenever the stone is smaller than the measuring unit then the measuring procedure can "overlook" this stone.

Thus the function

$$L = f(U) \qquad (A\ 2.6)$$

is non-increasing. For a realistic coastline it is purely decreasing.

The following specification of the function (*A 2.6*)

$$L(U) = K\ U^{(1-D)} \qquad (A\ 2.7)$$

was based on empirical observation. However, it can be formally proven using an analysis which is analogous to the entropy specification.

So, in this example the measured coast length is not a constant. It is a function of the measuring unit used (see (*A 2.4*)). Therefore the coast length itself cannot be used to provide an unambiguous characterization of the coast. From equation (*A 2.7*) it can be seen that according to fractal analysis two parameters are needed to characterize the coast, namely K and D. These can be used to specify the properties of the coast.

The parameter D is a measure of the degree of "roughness" of the object being measured. Under certain conditions, which are usually valid in the cases of interest here, this parameter corresponds to the Hausdorff dimension. The Hausdorff dimension of smooth lines is equal to one i.e. to its natural dimension. A topological (natural geometric) dimension, D_T, of an i-dimensional set is

$$D_T = i. \qquad (A\ 2.8)$$

However, non-smooth lines have

$$D_H \geq D_T, \tag{A 2.9}$$

where D_H is the Hausdorff dimension. The inequality (*A 2.9*) is correct for any *n*-dimensional set.

The fractal dimensionality D (*A 2.7*) is under certain conditions identical with the Hausdorff dimension (*A 2.9*).

$$D_H = D \tag{A 2.10}$$

The fractal dimension can be easily evaluated provided the relation L–U is known quantitatively (see (*A 2.7*)):

$$\log(L) = \log(K) + (1 - D)\log(U). \tag{A 2.11}$$

Using (*A 2.11*) it is easy in $\log(L)$, Y-, and $\log(U)$, X-, axes to evaluate D since it is a linear function:

$$Y = C + (1 - D)X, \tag{A 2.12}$$

where C is a constant. The key problem is therefore to evaluate quantitatively the relationship (*A 2.7*).

FRACTAL SCREENING

Suppose a black and white TV camera is used to observe a two dimensional object. This object is recorded from several different camera-object distances. The goal of these observations is to gradually eliminate object details. The camera sensitivity is constant, so, as the camera-object distance increases, only more and more significant features are recorded (are distinguishable). As a result of these records the following set of pairs is obtained:

$$(T_i, S_i)\ i = 1, 2, \ldots, n \tag{A 2.13}$$

where T_i is the *i*-th camera-object distance, S_i is the *i*-th TV image.

Let us suppose that there are just black and white pixels on the TV screen. Each screen record S_i contains a set of black and a set of white pixels:

$$S_i = (B_i, W_i)\ i = 1, 2, \ldots, n, \tag{A 2.14}$$

where B_i, (W_i) is the set of black (white) pixels on the *i*-th image. An isolated black pixel i.e. a detail disappears if the distance T (*A 2.13*) increases. A substantial feature represented by several pixels can be recorded even from a large distance. However, some details of this feature will gradually disappear.

Suppose the object under study is again a coast line. Then the length of the "coast" on the i-th screen image is directly proportional to the ratio:

$$L_i = \mathrm{card}(A_i)/z , \qquad (A\ 2.15)$$

where card(A) is the cardinality of the set A and z is the total number of screen pixels.

A simple commonsense analysis can again be usefully applied to show that:

$$\text{if } T_i < T_{i+1} \text{ then } L_i > L_{i+1} . \qquad (A\ 2.16)$$

As the camera-object distance is changed, so the camera cannot identify the same amount of detail on the screen. The total number of black pixels (see (*A 2.14*)) will gradually decrease as the camera-objectiv distance increases:

$$B_i > B_{i+1} . \qquad (A\ 2.17)$$

However, the total number (black plus white) of pixels is constant and equal to z (see (*A 2.15*)).

Therefore, from equation (*A 2.15*), the observed coast length decreases with increasing object distance (see (*A 2.3*)):

$$L_i > L_{i+1} . \qquad (A\ 2.18)$$

The elimination of detail in an m-dimensional space is strictly analogous to the above two-dimensional case.

A simple and efficient procedure for evaluating the fractal dimension D (*A 2.11*) of a realistic (extensive, vague and inconsistent) knowledge base is needed. The difficulty is that any realistic quantitative knowledge base (e.g. a set of fuzzy conditional statements) can have more than 100 variables. This is much too complex a situation for a mathematically exact analysis, and so some approximation technique must be used in order to obtain a computationally tractable algorithm.

The development of any tractable algorithm based on equation (*A 2.11*) needs a precise specification of the length and measuring unit (or threshold value) analogies. What is required is some mechanism for evaluating appropriate values L and U to insert into the above described algorithm. The problem will be choosing suitable analogies which are both simple and expressive at the same time.

Screening of Set of Fuzzy Conditional Statements

In principle it is possible to screen all those knowledge bases where a difference can be made between detail and non-detail. The specifications of the screening algorithms differ according to the characteristic of the knowledge bases. Nevertheless once a specification of detail as a function of threshold level is established then the procedure is straightforward.

Any measurable knowledge base is suitable for the application of fractal analysis. As a relatively simple problem a set of fuzzy conditional statements is studied.

The knowledge base is represented by a set of fuzzy conditional statements:

$$\text{if } A_i \text{ then } B_i \quad i = 1, 2, \ldots, m, \tag{A 2.19}$$

where

$$A_i = A_{i,1} \wedge \ldots \wedge A_{i,n} . \tag{A 2.20}$$

Let

$$r_i = A_{i,1} \wedge \ldots \wedge A_{i,n} \wedge B_i \tag{A 2.21}$$

represent the conditional statement (*A 2.19*) as an $(n + 1)$ dimensional fuzzy set.

Non-Structural Fractal Analysis

The simplest variant is studied here, namely where the mutual connections among statements are not taken into consideration. Roughly speaking only "volumes" of statements are considered and not their layout. In this case every one-dimensional fuzzy set $A_{i,j}$ in the n-dimensional fuzzy set A_i, see (*A 2.20*), contributes to an n-dimensional volume through differences (see Fig. 1):

$$d - \text{a zero } \alpha\text{-cut i.e. } \alpha = 0 \tag{A 2.22}$$

$$c - \text{b one } \alpha\text{-cut i.e. } \alpha = 1 \tag{A 2.23}$$

and not by their absolute values. An α-cut is a conventional set (an interval in the one-dimensional case) of the elements which have their grades of membership higher than the given α. In the example shown in Fig. 1, the zero α-cut is the interval of length $(d - a)$.

A length L, as defined in (*A 2.3*), is a measure which corresponds to a one-dimensional object (line). A fractal analysis of an $(n + 1)$-dimensional fuzzy model (*A 2.21*) needs an $(n + 1)$-dimensional volume V.

An α-cut volume of the n-dimensional fuzzy set A_i (*A 2.20*) and $(n + 1)$-dimensional r_i (*A 2.21*) are as

$$\alpha_V (A_i); \quad \alpha_V (r_i) . \qquad (A\ 2.24)$$

For $\alpha = 0$, we have from Eq. (*A 2.21*):

$$0_V(r_i) = (\prod (d_{Ai,j} - a_{Ai,j}))\, (d_{Bi} - a_{Bi}) , \qquad (A\ 2.25)$$

where $d_{Ai,j}$ is point d (see Fig. 1) of the grade of membership function of the one-dimensional fuzzy set $A_{i,j}$, see (*A 2.20*). The same notation is used for points b, c, d (see Fig. 1).

For $\alpha = 1$, we have:

$$1_V(r_i) = (\prod (c_{Ai,j} - b_{Ai,j}))\, (c_{Bi} - b_{Bi}) . \qquad (A\ 2.26)$$

The general expression for $\alpha_V(A(i))$, for $0 < \alpha < 1$, can be obtained from a "combination" of equations (*A 2.25*) and (*A 2.26*):

$$\alpha_V(r_i) = 0_V(r_i)\, (1 - \alpha)^n + 1_V(r_i)\, \alpha^n . \qquad (A\ 2.27)$$

The formula (*A 2.27*) defines the volumes for all m sets r_i (see (*A 2.19*)) of conditional statements.

The following simple algorithm is used for detail elimination:

$$\text{if } \alpha_V(r_i) < U \text{ then } \alpha_{VU}(r_i) = 0 \text{ else } \alpha_{VU}(r_i) = \alpha_V(r_i)\ i = 1, 2, \ldots, m , \qquad (A\ 2.28)$$

where U is a threshold volume which is used as a sort of n-dimensional measuring unit (see (*A 2.2*)).

The total volume of the fuzzy model (*A 2.19*) is:

$$T\alpha_{VU} = \sum \alpha_{VU}(r_i) . \qquad (A\ 2.29)$$

The measured volume is a function of U and α

$$T\alpha_{VU} = g\,(U, \alpha) \qquad (A\ 2.30)$$

in an analogous way to the function (*A 2.6*).

The function (*A 2.30*) enables the evaluation of the fractal dimension D (see (*A 2.7*), (*A 2.12*)) with α as a parameter (see Fig. 1), that is,

$$D = f(\alpha) . \qquad (A\ 2.31)$$

Note that the total volume $T\alpha_{VU}$ (*A 2.29*) is not "homogeneous" from the point of view of a dimensional analysis. Different variables may have different dimension (e.g. kg, °C, etc.). The simplest way to take account of this is to apply weight to the different variables:

$$W_1, W_2, \ldots, W_n, W_{n+1}, \qquad (A\ 2.32)$$

where W_{n+1} is the weight of the "dependent" variable B (see (*A 2.19*)).

We then have a weighted total α-volume (*A 2.29*). For $\alpha = 0$ (see (*A 2.22*), (*A 2.25*)):

$$W\,0_V(r_j) = \left(\prod_j W_j(d_{A(i,j)} - a_{Ai,j})\right) \times W_{n+1}(d_{Bi} - a_{Bi})\,. \qquad (A\ 2.33)$$

And for $\alpha = 1$ (*A 2.23, A 2.26*):

$$W\,1_V(r_j) = \left(\prod_j W_j\,(c_{A(i,j)} - b_{Ai,j})\right) \times W_{n+1}(c_{Bi} - b_{Bi})\,. \qquad (A\ 2.34)$$

If we let

$$V_1, V_2, \ldots, V_m \qquad (A\ 2.35)$$

be the weights attached to the conditional statements (see (*A 2.19*)), then the total weighted volumes are:

$$T\,0_V = \sum W\,0_V(r_i)\,V_i \qquad (A\ 2.36)$$

$$T\,1_V = \sum W\,1_V(r_i)\,V_i\,. \qquad (A\ 2.37)$$

Fractal Optimization

Using the same optimization philosophy as presented in (*A 1.62*) it is possible to minimize the level of chaos represented by the fractal dimensions. There can be different sets of independent variables X (see (*A 1.62*), (*A 1.63*)). The obvious candidates are fuzzification coefficients f (see (*A 1.22*)).

A very promising application of fractal optimization is a problem which can be specified as follows:

objective function = integrated modification of the knowledge base (*A 2.19*) (*A 2.38*)

constraint $D = i$ (see (*A 2.8*)) . (*A 2.39*)

Such optimization problems can increase efficiency of complex upgrading activities considerably.

Fractal Reconstruction

Information intensity of the conventional formal tools (e.g. statistics) is the severest problem of any realistic analysis. An information intensive meta heuristics must be used to substitute the missing knowledge items[37].

The meta heuristics must be based, directly or indirectly, on commonsense[38]. They must represent this commonsense. This is the only general source of knowledge which is available in engineering practice. One possible meta heuristic is based on the idea of fractal reconstruction. The extreme type of fractal reconstruction is:

– reconstruct a knowledge base with prescribed fractal dimension.

A more flexible meta heuristic is:

– generate a knowledge base which when merged with a known knowledge base gives a knowledge base with the prescribed fractal dimension.

Many different meta heuristics can be presented. Useful analogy and therefore very promising inspiration is offered by different algorithm for treatment of numerical pictures[39].

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